MES.P12

An echo top estimation using vertical interval interpolation

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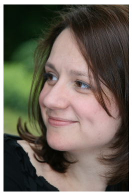

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1. Introduction

The echo top parameter derived from radar observations is widely used to identify storms with large vertical extensions that may point to potential severe weather such as hail (Waldvogel et al., 1979) and wind gusts. Unfortunately, the limitations of existing echo top calculation methods were recognized in different studies (e.g., Atlas et al., 1963; Delobbe and Holleman, 2006; and Lakshmanan et al., 2013).

In the course of this particular study we define the echo top as the maximum height $H_{i,h}$ (in km AMSL) related to the measured echo $Z_{i,h}$ (in dBZ) at the *i*-th point of the measurements domain

$$EchoTop_i = \max\left(\left\{H_{i,h} : Z_{i,h} \geq \tau\right\}\right).$$

where τ represents the different threshold values and *h* runs through a number of discrete values corresponding to scanning elevations.

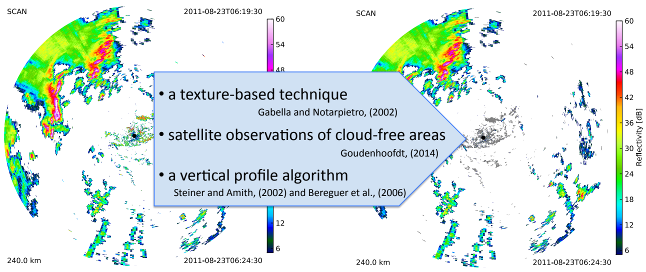
The assignment of the height at which echo top appears and the error related to the measurement of the reflectivity itself are two possible sources of echo top estimation uncertainty. One solution is to assume a locally linear variation in the vertical reflectivity profile near the cloud top as was suggested in Lakshmanan et al., (2013).

The idea of this study is **to test a technique that estimates the echo top exploiting not only the vertical but also the horizontal variation of reflectivity measurements.**

The assumption of local linearity is relaxed by allowing also non-linear variation of reflectivity vertically and horizontally. A vertical segment interpolation technique is a suitable solution for this problem because it deals with measurement uncertainties in a very natural way.

2. Radar Measurements

In the study data of the C-Band (5.62 GHz) weather radar in Wideumont (Belgium) are used. The scan we use in this study has 10 elevations between 0.5° to 17.5° and is performed every 15 minutes. The samples are collected every 500 m in range. The maximum range of the radar measurements is 240 km. The data are coded with a resolution of 0.5 dB. No Doppler filtering is used, but several post-processing corrections are applied for clutter removal:



3. Interpolation Method

The generalized rational multivariate interval interpolation technique (GRMIIT) uses generalized rational functions (GRF) of a finite dimensional real vector $\bar{x}$ of the form:

$$r_{n,m}(\bar{x}) = \frac{p_{n,m}(\bar{x})}{q_{n,m}(\bar{x})} = \frac{\sum_{j=0}^{n} p_j b_j(\bar{x})}{\sum_{k=0}^{m} q_k b_k(\bar{x})} \qquad (3.1)$$

with coefficients p_j, q_k and multivariate basis functions, formed by Chebyshev polynomials of the first kind. For the normalization, one of the coefficients can be fixed and the rational function (3.1) will have n+m+1 degrees of freedom.

In GRMIIT we consider a set of *s+1* measured values f_i, *i=0,…,s* with an uncertainty intervals $F_i = [\underline{f_i}, \overline{f_i}]$. For the set of intervals F_i we search for a GRF of the form given in (3.1) that satisfies

$$\underline{f_i} \leq r_{n,m}(\bar{x}_i) \leq \overline{f_i}, \quad i = 0,\dots,s \qquad (3.2)$$

where $n + m \leq s$.

Assuming q_k>0, *i-0,…,s* and linearizing (3.2) we obtain the homogeneous system of linear inequalities

$$\begin{cases} p_{n,m}(\bar{x}_i) - \underline{f_i} q_{n,m}(\bar{x}_i) \geq 0 \\ -p_{n,m}(\bar{x}_i) + \overline{f_i} q_{n,m}(\bar{x}_i) \geq 0 \end{cases} \quad i = 0,\dots,s \qquad (3.3)$$

Let us denote by $A_{n,m}$ the *(2s+2)x(n+m+2)* matrix corresponding to the inequalities in (3.3),

$$A_{n,m} = \begin{pmatrix} b_0(\bar{x}_0) & \cdots & b_n(\bar{x}_0) & -\underline{f_0} b_0(\bar{x}_0) & \cdots & -\underline{f_0} b_m(\bar{x}_0) \\ \vdots & & \vdots & \vdots & & \vdots \\ b_0(\bar{x}_s) & \cdots & b_n(\bar{x}_s) & -\underline{f_0} b_0(\bar{x}_s) & \cdots & -\underline{f_s} b_m(\bar{x}_s) \\ -b_0(\bar{x}_0) & \cdots & -b_n(\bar{x}_0) & \overline{f_0} b_0(\bar{x}_0) & \cdots & \overline{f_0} b_m(\bar{x}_0) \\ \vdots & & \vdots & \vdots & & \vdots \\ -b_0(\bar{x}_s) & \cdots & -b_n(\bar{x}_s) & \overline{f_s} b_0(\bar{x}_s) & \cdots & \overline{f_s} b_m(\bar{x}_s) \end{pmatrix} \qquad (3.4)$$

and by $A^{(j)}_{n,m}$ the *j*-th row of the matrix (3.4). It was shown in Salazar Celis et al., (2007) that a robust solution $r_{n,m}$ of (3.3) can be computed from the following quadratic programming problem:

$$\arg\min_{\bar{c}\in R^{n+m+2}} \|\bar{c}\|_2^2 \quad \text{subject to} \quad A^{j}_{n,m}\bar{c} - \delta\|A^{j}_{n,m}\|_2 \geq 0, \qquad (3.5)$$

with $\bar{c} = (p_0,\dots,p_n,1,q_1,\dots,q_m)^T$ and $\|\ \|_2$ denoting the Euclidean norm. The real value $\delta > 0$ is set to the inverse of the condition number of the matrix $A_{n,m}$ (Cuyt et al., 2014).

We search for a suitable n and m through the sequence (0,0), (1,0), (0,1), (2,0), (1,1), (0,2), (3,0), (2,1), (1,2), (0,3), (4,0), … For *n+m=s*, a solution is guaranteed to exist.

2D vertical segment interpolation of echo top

The available heights $H_{i,(h-1)}$ and $H_{i,h}$ corresponding to the closest reflectivities $Z_{i,(h-1)}$ and $Z_{i,h}$ enclosing the threshold value $\tau, Z_{i,(h-1)} \geq \tau \geq Z_{i,h}$, are extracted for each *i*-th point of the measurements domain. The heights $H_{i,(h-1)}$ and $H_{i,h}$ form the vertical intervals F_i.

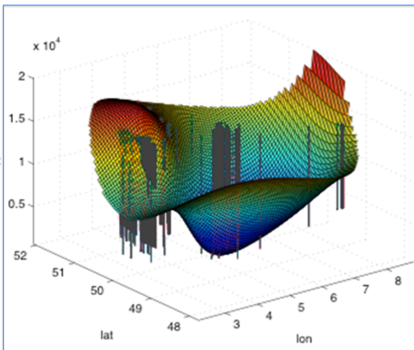


Figure 1: The interpolating GRF with 1506 vertical intervals

4. Results and discussion

The volumetric measurements of Wideumont's weather radar are transformed into a set of uncertainty intervals for the height of a given echo top threshold of 45 dBZ. The results of the interpolating height for 23 August 2011 (at 0634 UTC) are presented in Figure 1. The GRMIIT returns a $r_{n,m}$ function with *n+1=5* numerator coefficients and *m+1=15* denominator coefficients. The resulting echo top function values for these points are shown in Figure 2.

To conclude, in this work we have presented a generalized rational multivariate interval interpolation technique and showed the preliminary results of its application to the echo top estimation. As can be observed in the figures, some residual artifacts due to ground clutter close to the radar influence the echo top interpolation function. This emphasizes the importance of the quality control of the radar measurements. The obtained interpolating function is only a rough estimate of the true echo top values. For more realistic results data from several radars could be used to decrease the maximum size of interpolating intervals.

The application of GRMIIT may be extended to the interpolation of the 3D radar reflectivity field. In such a way the vertical segments represent the uncertainty in the reflectivity measurement at each point of the radar volumetric data. From the interpolation function $r_{n,m}(\phi,\lambda,h)$ the discrete level surface $r_{n,m}(\phi,\lambda,h) = 45$ provides a more precise echo top estimation. It will also allow us to better assess the effects of both types of uncertainties (in height and in reflectivity) in the radar measurements.

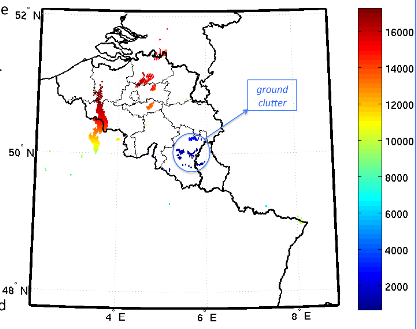


Figure 2: EchoTop 45 dBZ (in m AMSL).

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